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THE APPLICATION OF THE PLANE GRATING TO THE DETERMINATION OF THE INDEX OF REFRACTION OF A GAS WITH VALUES FOR AIR FROM λ 2500 TO λ 6500.

DISSERTATION
SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES
OF THE JOHNS HOPKINS UNIVERSITY
IN CONFORMITY WITH THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY.

BY

ROBERT WILLIAM DICKEY.

BALTIMORE

1916.

172,409

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Introduction.

There are many methods applicable to the determination of the index of refraction of a gas for wave-lengths in the visible spectrum and near ultra-violet spectrum. Any method to be of universal use must give results in the extreme ultra-violet, which are accurate to the same degree as those in the visible region. It was the purpose of the present investigation to study the use of the plane grating in extending as far as possible into the ultra-violet region the dispersion curve of a gas. The test of any method is the value of the results derived by it, consequently the index of refraction of air has been obtained for different wave-lengths between 2493 and 6412 A.U. A careful study of the possibilities and difficulties of the method has been made.

Historical.

As the problem of finding the index of refraction is an old one, it has been undertaken by a great many investigators. A few of the most important methods will be reviewed along with the results found:

The dispersion of gases was first studied by Ketteler(1), who placed between the plates of a Jamin interferometer two tubes of equal length, filled with the gas under investigation. The fringe shift was

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(1) "Pogg. Ann." vol. 124, 390-406, 1865.

observed for a definite change in pressure of the gas in one tube. This method gave absolute values for the index of refraction but the range of observations was limited by the difficulty of obtaining efficient sources of monochromatic light. He obtained the index of refraction of yellow sodium light for air, carbon, dioxide, hydrogen, sulphur dioxide and cyanogen.

Lorenz(1) working with a similar apparatus, repeated some of Ketteler's measurements and determined the refractive indices of air, hydrogen, oxygen and nitrogen for the sodium and lithium light. He passed the air through proper solutions to free it of moisture and carbon dioxide.

Mascart(2) divided a beam of light into two parts, passed these parts through two tubes filled with the gas under investigation, recombined them and passed them through a spectroscope. If the pressure in the two tubes be different, Talbot bands appear, due to a change in the optical path. He measured the absolute value of the index of refraction of air and chlorine for sodium light and the relative values for the four cadmium lines. Perreau(3) used the actual tubes of Mascart, but discarded the method involving Talbot bands and obtained

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(1). L.Lorenz."Ann der Phys. und Chem." N.F. 11, 70, 1880.

(2). Mascart. "Comptes Rendus, 86, 321, 1878.

(3) . Perreau. "Ann de Ch. et de Physique" 7, 289, 1896.

a banded spectrum by analysing white light which had passed through a Jamin refractometer. Kayser and Runge(1) placed a hollow prism between a photographic plate and a large concave grating and measured the displacement of various spectral lines when the pressure of the air in the prism was changed. They worked between λ=5630 and λ=2360 with a pressure change of 10 atmospheres.

Rentschler(2) used a Fabry and Perot interferometer and a concave grating and determined the indices of refraction of air, nitrogen, oxygen, carbon dioxide and carbon monoxide from λ=5769 to λ=3341. The interferometer plates were silvered, and as the reflecting power of silver is low between λ=2280 and λ = 3341 his results were not carried below λ = 3341.

The most recent work in the visible spectrum is that of C. & M. Cuthbertson. They used a Jamin refractometer and determined the indices of refraction of air, oxygen, nitrogen, hydrogen(3), neon(4), krypton, xenon, helium, and argon(5), between λ = 6563 and λ = 4861.

Very recently an excellent investigation of this problem has been carried out by Miss Howell(6). By using a Fabry and Perot interferometer

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(1). Kayser and Runge."Annalen der Physik" 50, 293, 1893.
(2). Rentschler. "Astrophys. Jl." 28, 345, 1908.
(3). C. and M. Cuthbertson."Proc. Roy. Soc." 83, 151, 1909.
(4) " " " " " " " 83, 149, 1909.
(5) " " " " " " " 81, 440, 1908.
(6). Miss Howell. "Phys. Review" VI, 81, 1915.

(with plates nickeled by cathode discharge), in connection with a quartz spectrograph, she has extended the dispersion curve of air to $\lambda = 2652$ and those of hydrogen, oxygen, and carbon dioxide to $\lambda = 2753$. Dispersion formulae for these gases were calculated from the results obtained. This method is superior to that in which a Jamin refractometer is used, in that the temperature can be more easily controlled and it is not necessary to count the number of interference bands going by for a change of pressure. This latter advantage makes it possible to work in the ultra-violet region.

The method here described consists in determining the index of refraction of a gas by finding the ratio of the wave-lengths in vacuum and in the gas, by means of a Rowland plane grating, using a Littrow type of mounting. The general idea involved is not new, for Osborne and Lester(1) used this method for the determination of the index of refraction of a liquid. Its application to a gas is new and there are many details in which this method differs from the one mentioned above.

Theoretical Considerations.

If v_0 is the velocity of light waves in the pure ether, v, the velocity in a definite gas, then the index of refraction of the gas with reference to the ether is defined as the ratio of v_0 to v. Calling the index of refraction n, we have

$$n = \frac{v_0}{v} = \frac{\nu \lambda_0}{\nu \lambda}$$ where ν is the frequency corresponding

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(1)Osborne and Lester "Physical Review", 35, 210, 1912.

to the wave-length λ.

$$\therefore\ n = \frac{\lambda_o}{\lambda} \text{ and } n-1 = \frac{\lambda_o - \lambda}{\lambda} = \frac{\Delta\lambda}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The wave-lengths in air, λ, are very accurately known, so it is necessary to measure the wave-lengths, λ_o, in vacuum, or rather to obtain the change in wave-length, $\Delta\lambda$, of the light for a known change in pressure of the gas. To accomplish this, the fundamental principle of the grating is made use of. The condition that we obtain a bright line of m^{th} order, when white light is allowed to fall upon a plane grating, is that the path difference between rays reflected from adjacent grooves shall be $m\lambda$, where λ is the wave-length of the line in question. The equation for the Littrow mounting is

$$m\lambda = s\,(\sin\theta + \sin i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where θ is the angle of incidence of the light and this is kept constant, and i is the angle of the diffracted beam - i differs very slightly from θ. If the grating is surrounded by a gas, let the path difference for some particular line in the spectrum be $m\lambda_1$. Then when the same grating is placed in a vacuum, the path difference between adjacent grooves for the same line will be $m\lambda_2$ according to equation (1) where λ_2 is larger than λ_1. Since θ is constant in equation (2) the change in λ is accompanied by a change in i and a shift of the line is observed. If a photographic plate is placed

in the focal plane of the observing telescope, this shift can be recorded by making two exposures, one with the grating in vacuum and the second with the grating in the gas, keeping the grating fixed during the two exposures.

Apparatus and Method.

The Littrow form of spectrometer (Fig. I) was used, in which the collimator and observing telescope are united. The plane grating, mounted on a heavy iron base-plate, receives the light from the objective and returns the diffracted beam along the same path as the incident light or along a path which makes a very small angle with that of the incident light.

To avoid instrumental shifts due to the possible relative motion of the parts of the apparatus it was necessary to mount the apparatus as rigidly as possible. Two 3-inch iron I-beams, 10 feet in length, were fastened together and parallel to each other so that the flanges gave a firm horizontal surface on which to mount the various parts. At one end a cast iron base-plate, 8.5 inches in diameter and 1.25 inches thick, was mounted and insulated from the I-beams by hard rubber supports so as to prevent as much as possible the transference of heat to or from the plate. At the centre of this plate the grating-holder was attached with the proper adjustments. Just behind the

Apparatus.

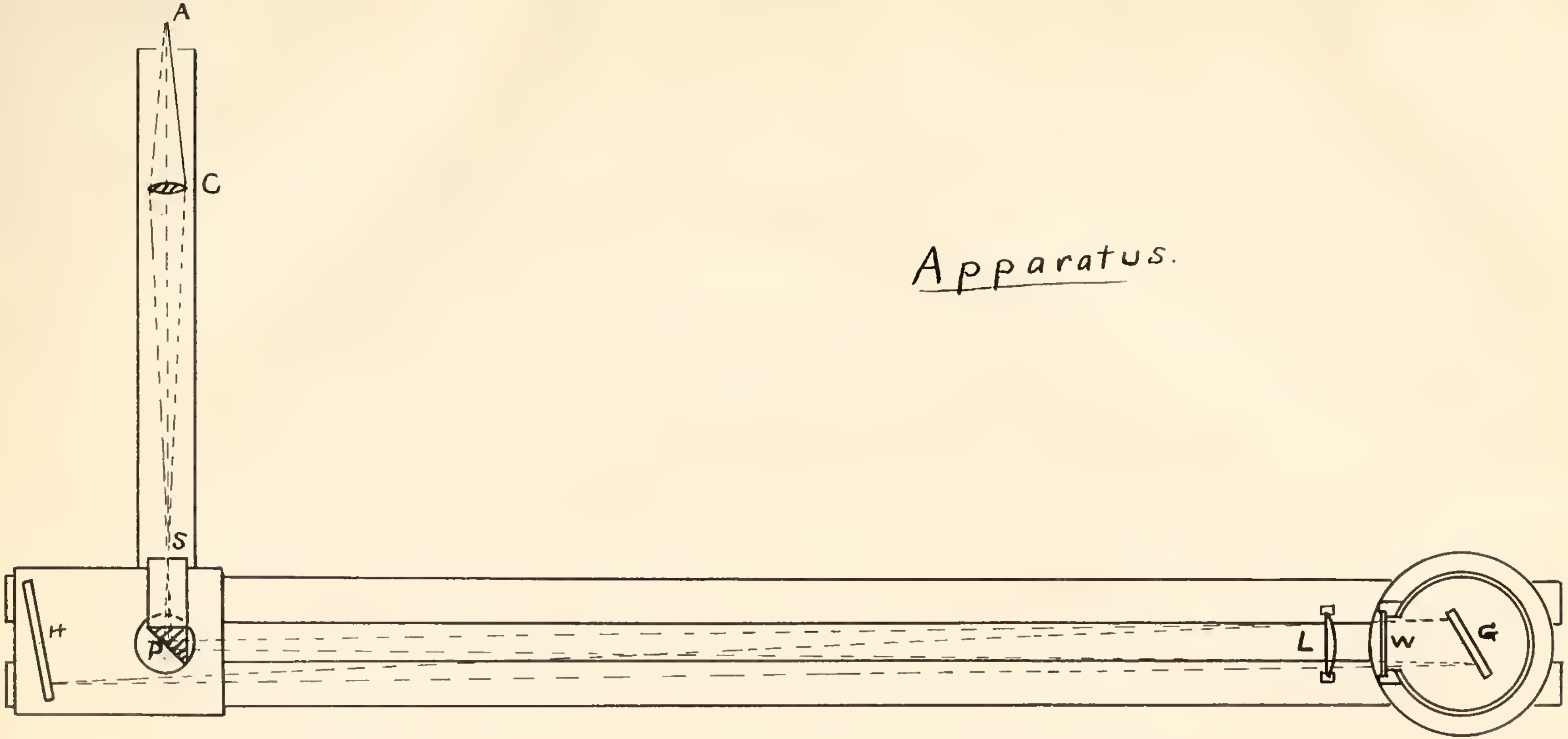

Fig. 1. (Top view)

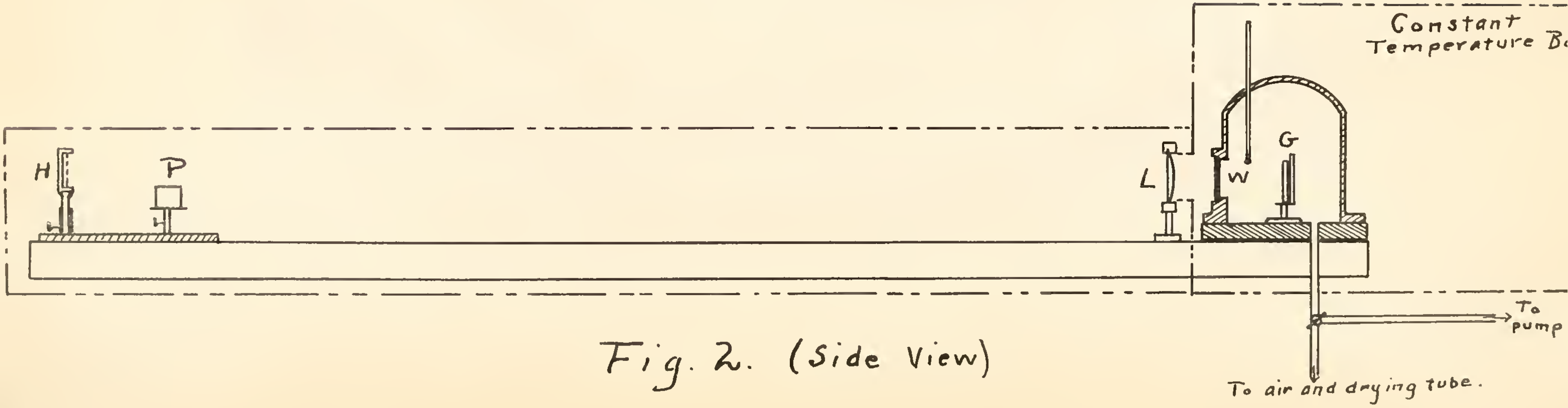

Fig. 2. (Side View)

grating holder the connection to the pump and outside air was made as shown in Fig. 2. At the other end of the I-beams an iron plate, carrying the slit, reflecting prism and photographic plate holder, was fastened. The collimator lens was mounted on a slide fastened between the I-beams. In focussing the apparatus for any definite region of the spectrum the plate holder and slit were kept fixed and the collimator lens moved. A heavy brass bell jar, 5 inches inside diameter, 5.5 inches deep with walls 3/8ths of an inch thick and a heavy flange at the bottom, covered the grating. The front was arranged so that a quartz window could be inserted and made air-tight. The opening for the window was 1 x 1.5 inches. Through a hole in the top of the bell-jar a thermometer, reading to tenths of a degree, was inserted, in order to determine the temperature of the gas under investigation.

A Rowland 2.5 inch grating, ruled with 15000 lines to the inch, was used. As more accurate results were obtained by using the higher orders, a grating was selected which gave a bright third and fourth order. On account of the high coefficient of expansion of speculum metal every precaution had to be taken to keep the temperature of the grating constant during a series of exposures. A heating coil wound on the inside walls of a large wooden box (16 x 22 x 30 inches), surrounding the bell-jar helped to keep the temperature constant.

Then the massiveness of the bell jar tended to oppose any rapid changes in the temperature of the inside. This problem of temperature has proven a very serious trouble and it will be discussed more fully in a later paragraph.

The entire optical system, consisting of a small condensing lens, C, a totally reflecting prism, P, the collimator lens, L, and the plane-parallel plate, W, was made of quartz. As a source of light the Pfund iron arc was used. This was operated on the 110-volt circuit with 5 or 6 amperes, except in the red region of the spectrum where the current was increased. An image of the arc was thrown on the slit, S, by means of the condensing lens, C, and a larger image was projected on the wall of the room at a distance of 15 feet so as to be certain that the same part of the arc was being used in the two exposures, with and without the gas. The observations were recorded on photographic plates, which were developed with rodinal.

The general procedure in obtaining observations was to pump out the bell-jar until the pressure was a small fraction of a millimeter of mercury, take an exposure on the photographic plate, then let in the air through drying solutions and make a second exposure on the same plate, being very careful not to disturb it between exposures. In order to obtain all results under exactly the same conditions a scheme for the

operations was adopted and adhered to throughout the entire work. The importance of this procedure will be touched upon in discussing the temperature conditions. The scheme was to pump out and let in the air at intervals of twenty minutes, taking the first exposure so that the middle of the exposure came three minutes before letting in the air and the second exposure three minutes after letting in the air. The exposures for different regions of the spectrum varied from 15 seconds to 5 minutes, so it was necessary to adopt the three minute intervals mentioned above.

The third order spectrum was used between λ = 6400 and λ = 4300; the fourth order from λ = 4100 to λ = 2700; and the third order for the extreme ultra-violet observations. Even though the overlapping spectra were not in focus, much trouble was experienced by the fog produced on the plates. By the proper selection of absorbing solutions, this was avoided to a certain extent. A cell containing the solution was placed in front of the slit of the spectrometer. The spectrum of the iron arc is most intense in the region of λ = 3900, and it was necessary to find a solution which would absorb this region and transmit lower regions in the ultra-violet. For this purpose chromium chloride or nickel sulphate in a cell with quartz faces was found to be very good. The former has an absorption band extending from λ = 3800 to λ = 4500 and the latter a band from λ = 3700 to λ = 4250, a second

band beginning at λ = 2350. The cell was about 1 cm. in thickness and filled with dilute solutions of the above compounds.

In the design of the apparatus any mechanism for setting the grating without removing the bell-jar was purposely avoided, for it was thought that such an arrangement might cause an instrumental shift in the grating when the pressure in the enclosure was changed. The constant temperature box was arranged so that it could be removed in sections without injury to the heating coil and then the bell-jar was removed from the base plate. The grating was set at the desired angle by means of a telescope and scale, using the grating as a mirror. When the bell-jar was replaced over the grating it was necessary to make the plane-parallel quartz window perpendicular to the incident light from the lens. It was by accident that a very fine means of doing so was found. A certain percentage of the incident light is reflected by the window and this is brought to a focus in the focal plane of the collimator lens, thus producing an image on the photographic plate. By regulating the tilt of the grating and the height of the plate the image could be made to fall just below the spectrum on the plate. In the discussion of the observations it will be seen that the location of this image has an important bearing on one of the corrections. In adjusting the bell-jar this

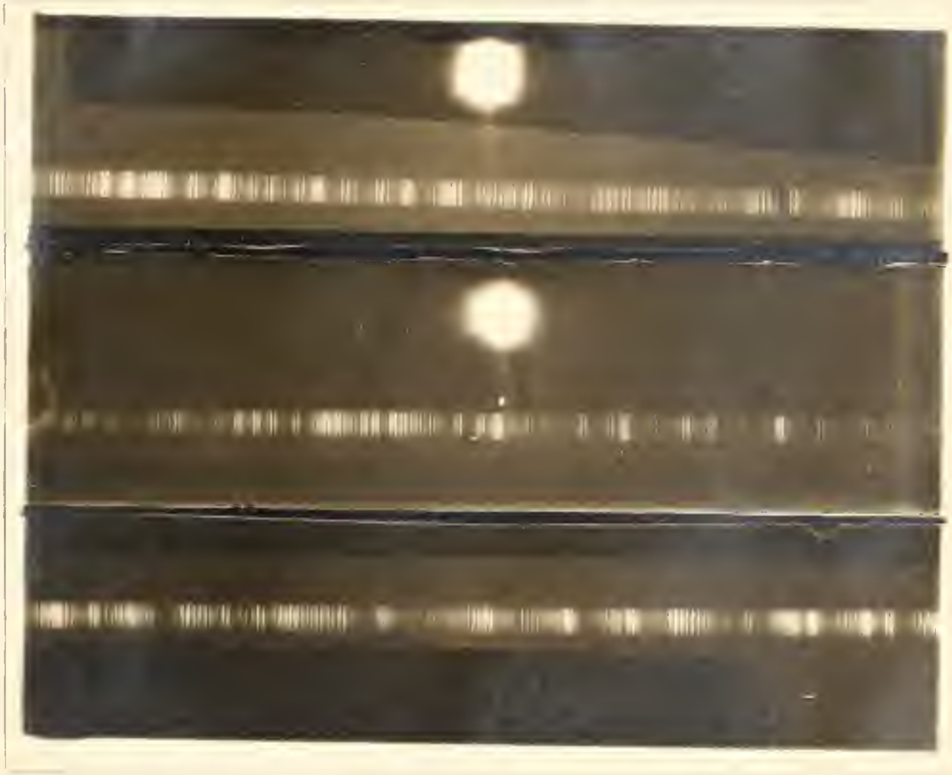

Fig. 3.

reflected beam was made to pass directly above the point on the prism from which the incident light proceeded. A reference to Fig. 3 will show this reflected image.

The heating coil in the constant temperature box consisted of several hundred feet of fine german silver wire, two coils being connected in parallel to the 220 volt direct current circuit. The regulation of the temperature depended upon the expansion or contraction of a large quantity of mercury contained in an inverted U-shaped tube so as to give a large surface compared to the amount of mercury used. The battery connections of the relay circuit were made through a small capillary tube connected to the U-tube at the bend.

The air was freed from moisture by passing it through sulphuric acid and over phosphorous pentoxide. The pressure in the bell-jar was measured by a mercury manometer, the readings being reduced to mercury at 0°C. Readings of temperature were made to $\frac{1}{20}^{\circ}$ C. and pressure to one-tenth of a millimeter.

Temperature Considerations.

Preliminary experiments showed that the greatest difficulty to be overcome was the change in temperature of the grating between exposures, due to a thermodynamic increase or decrease in the temperature of the surrounding air when the pressure was increased or decreased.

The resultant change in the temperature of the air itself is negligible so far as the density of the air is affected, but the high coefficient of expansion of the metal of the grating causes change in the grating space, and this in turn causes a considerable shift of the spectrum lines for even a small fraction of a degree change in temperature. Upon reducing the pressure in the bell-jar to a fraction of a millimeter of mercury, a sudden drop of about 1°C was indicated by the thermometer whose bulb was inside the enclosure. Then the thermometer began to rise and after fifteen or twenty minutes apparently reached a steady state. After waiting ten minutes longer the pressure of the air was raised to atmospheric pressure and the thermometer showed a rise of about 1°C above ths steady state point. Then it slowly came back to the original value.- A rough calculation indicated that the actual change in the temperature of the air alone was in the neighborhood of 100°C, but the greater the heat capacity of the surrounding materials, the less would be the change in their temperatures. From these numerical considerations we should expect that the changes in the temperature of the grating would be sufficient to make a large error in the final results.

If one could tell when the grating reached a steady state the difficulty would be overcome, but it is evident that there is a considerable temperature lag in the grating with reference to the ther-

mometer. An attempt to settle this point was made by the use of a thermo-couple, one set of junctions being insulated and imbedded in a brass block attached to the edge of the grating, the other set being maintained at $0^{\circ}C$. It was at once recognized that the effect shown by the galvanometer was not that alone due to the heating or cooling of the grating , but in addition an effect due to the conduction of heat to or from the junctions by the leads which had to pass unprotected through the air in the enclosure. Any change in the temperature of the air would be conducted to the junctions and produce a deflection of the galvanometer. Although it was impossible to separate one effect from the other, nevertheless the attempts showed that even after an interval of one hour the grating was still changing in temperature. The thermo-couple was sufficiently sensitive to detect a change of $1/1000^{\circ}C$.

An attempt was then made to avoid the difficulty by lowering the temperature of the air before letting it into the bell-jar. The general scheme was to lower the pressure and allow the apparatus to remain for several hours or sufficient time for conditions in the bell-jar to become steady; then to cool the air below room temperature by an amount sufficient to counteract the thermodynamic rise, and let it pass into the bell-jar. A compact brass coil of heavy walled tubing of small inside diameter was connected in the circuit just below the opening into

the bell-jar. The coil was immersed in liquid air and allowed to remain so until it was thoroughly cooled. The air was passed slowly through this coil into the evacuated bell-jar, but the same rise in temperature, as indicated by the thermometer, was observed. This method had to be discarded.

Although this error due to temperature changes could not apparently be done away with, it was decreased to a certain extent by filling all the unnecessary space in the bell-jar with pieces of metal with high specific heats and large absorbing surfaces. This not only decreased the percentage of heat absorbed or given out by the grating, but it decreased the amount of air and consequently the total change in temperature due to the thermodynamic processes.

Finally the scheme of lowering and raising the pressure of the air at intervals of twenty minutes and taking all exposures three minutes before and three minutes after letting the air into the bell-jar was adopted. In this way the error due to temperature changes will be the same for all observations. The above cycle of operations was repeated several times before any observations were made. Some knowledge of this error was obtained by taking a series of observations at time intervals other than three minutes before and after letting in the air. As far as the shift of the spectrum line due to the change in density of the gas is concerned , the effect on this shift due to the small change

in temperature of the grating is negligible. So this density shift is practically constant for the exposures at intervals other than three minutes. But the amount of the additional shift due to grating temperature changes depends upon the time before and after, at which the two exposures are made. Thus if one plate is made taking an exposure 3 minutes before and 1 minute after letting in the air and a second plate 2 minutes before and 1 minute after, the difference between the shifts of any definite line on the two plates will give the shift due to the grating temperature change in the interval from 3 minutes before to 2 minutes before letting in the air. This was carried out for intervals of ten minutes before and after letting in the air taking observations for every minute. The results were sufficient to indicate that at the time adopted for the first exposure of a plate the temperature of the grating was rising and continued to rise receiving an additional impulse when the pressure of the air was increased. This rise continued through the time adopted for the second exposure and then began to fall passing through a zero value about the time set for the lowering of the pressure of the gas. The amplitude of the change for the interval of six minutes between the two exposures was about what was to be expected if the entire error was due to this temperature change. As the total change in the shift is only a few thousandths of a millimeter, it is asking too much of the method to

expect a determination of this temperature error to the same degree of accuracy as is required in obtaining the total shifts.

The only course left open was to take the mean value of the known values of the index of refraction of air for some line in the visible spectrum and refer all observations to this one value.

Results.

Observations were made at intervals of about 200 Å.U., three or four plates being made for each region. This gave a means of checking the measurements taken from a single plate. In every case tried, the values from two different plates were found to agree.

The position of the lines on the plates and their corresponding shifts were measured by a travelling microscope, the screw of which was made in the same way as the screws for the Rowland ruling machines. Measurements were made to 1/1000 mm. In cases where the lines on any one plate were numerous, only those best adapted to measurement were selected. Each line and its corresponding shift were measured on an average of six or seven times and the mean of the results taken.

Position Correction:- In the preliminary experiments it was noticed that the shift was not only horizontal, but vertical to a small degree. In looking for an explanation of this, it was found that a correction, depending upon the distance of the line from the center

of the plate, had to be made. This correction can be deduced with the aid of Fig. 4. The only essential parts to be considered are the quartz plate, W, which serves as a boundary between the outside air and the enclosure, and the photographic plate. If, when the pressure in space 2 is the same as that in space 1, the path of the ray from the grating, G, to the photographic plate is ABC, its path, when the pressure in 2 is less than that in 1, will be ABD due to the deviation of the ray as it passes from a rarer to a denser medium. At the center of the optical system this will not occur for the ray is normal to the quartz plate, W.

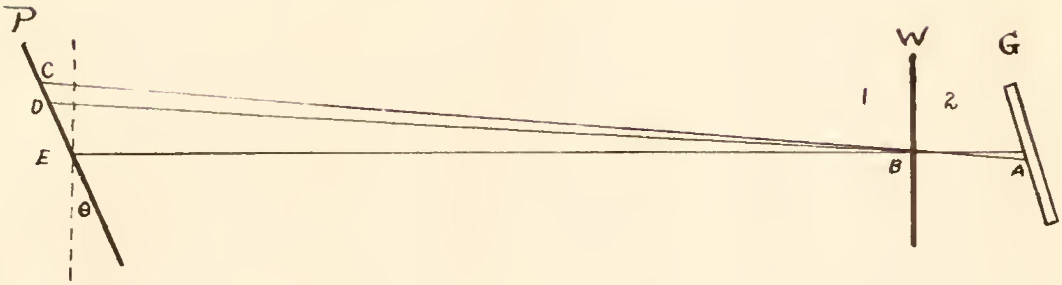

Fig. 4.

Let n' = the index of refraction of space 1 with reference to space 2 at the observed temperature and pressure. This value of n' is derived from measurements at the center of the photographic plate where no correction is necessary.

Let ℓ = distance ED, or distance of line from center of plate

Let θ = tilt of photographic plate

Then $CD = \left(\frac{n' \ell \cos \theta}{EB} - \frac{\ell \cos \theta}{EB} \right) EB$, approximately

$$= (n'-1)\, \ell \cos \theta$$

On account of this error the measured shifts taken from the half of the plate towards the red end of the spectrum are too small, and those of the other half too large. In referring the shifts to the center of the plate, the above correction had to be added to or subtracted from the measured values of the "red" end or "blue" end respectively of the plate. Obviously it was very necessary to determine the center of the photographic plate and this was done by means of the image of the slit reflected from the front surface of the quartz window. This has been discussed in the section on the apparatus and method.

The values of the shifts, after the above correction had been made, were reduced to standard conditions. As a result of many experiments the following relation between the index of refraction of a substance and its density,

$$\frac{n-1}{\rho} = \text{constant},$$

has been well established by many investigators.

Further, the shift for any definite wave-length is such that

$$\frac{n-1}{\Delta\lambda} = \text{const.}$$

Hence, putting $\Delta\lambda = s$, it follows that

$$\frac{s}{\rho} = \text{const.}$$

Or, expressing ρ in terms of pressure and absolute temperature

$$\frac{sT}{p} = \text{const.}$$

Hence, if s_1 is observed at temperature T_1 and pressure p_1 (expressed in mm. of mercury), the value of s which would have been observed if the temperature had been 0°C and the pressure 760 mm. is given by the formula

$$\frac{s\,273}{760} = \frac{s_1\,(273+t_1)}{p_1}$$

$$\text{or} \quad s = s_1\,\frac{760}{p_1}\,\frac{273+t_1}{273}$$

As the shifts reduced to standard conditions were given in millimeters, it was then necessary to reduce them to Ångström units. To do so the dispersion for each line of the plate had to be calculated. The positions of three lines, one at the center of the plate, x_0, and one near each end of the plate, x_1 and x_2 respectively, were carefully measured. It was assumed that any two wave-lengths with their corre-

sponding positions obeyed the following relation,

$$\lambda_0 - \lambda = A (x_0 - x) + B (x_0 - x)^2$$

in which A and B are constants, x_0 the position of λ_0 on the plate, and x the position of any other line λ. λ_0 is taken as near as possible to the centre of the plate. To obtain the values of A and B the three lines mentioned above were used. Differentiating the expression we have

$$\frac{d\lambda}{dx} = A + 2B (x_0 - x)$$

Then if x_0 is selected, the dispersion at any other point on the plate indicated by x is known from the above relations. All the shifts were then multiplied by the value of $\frac{d\lambda}{dx}$ for the corresponding value of x. The resultant values are the respective changes in the wavelengths of the different lines of the spectum as thepressure is changed by the amount observed. According to equation (1), n - 1 $= \frac{\Delta\lambda}{\lambda}$; hence the values of n - 1 could be calculated. The wavelengths of the lines were taken from Kayser's Handbuch.

A specimen of the results is given in the following table. This is one of the seventeen tables obtained.

I x(mm)	II λ(Å.U)	III S_2(mm)	IV S_1(mm)	V S(mm)	VI $\frac{d\lambda}{dx}$ $\frac{(\text{Å.U})}{\text{mm}}$	VII Δλ(Å.U)	VIII n-1	IX.
λ_1 256.188	4199.25	.5501	.5338	.5943	2.1285	1.2650	.0003012	For λ=4219.52 n-1 =.0003011
258.483	4204.15	.5506	.5349	.5956	2.1276	1.2671	.0003014	
259.964	4206.87	.5509	.5355	.5962	2.1256	1.2673	.0003012	
263.039	4213.82	.5505	.5360	.5968	2.1232	1.2671	.0003007	
264.243	4216.33	.5508	.5366	.5984	2.1223	1.2699	.0003012	
265.740	4219.52	.5514	.5376	.5966	2.1211	1.2696	.0003009	
277.985	4245.43	.5540	.5433	.6049	2.1115	1.2772	.0003009	
288.707	4268.93	.5565	.5486	.6108	2.1032	1.2847	.0003009	For λ = 4315.29 n-1 =.0003005
290.574	4271.95	.5562	.5488	.6111	2.1017	1.2843	.0003006	
295.643	4282.59	.5575	.5514	.6139	2.0978	1.2879	.0003007	
297.103	4285.60	.5567	.5510	.6135	2.0966	1.2863	.0003001	
301.239	4294.32	.5587	.5541	.6170	2.0934	1.2916	.0003008	
306.647	4305.63	.5590	.5558	.6188	2.0892	1.2928	.0003003	
307.818	4308.09	.5598	.5569	.6201	2.0883	1.2950	.0003006	
λ_0 311.262	4315.29	.5605	.5585	.6219	2.0856	1.2970	.0003006	
319.000	centre of plate							
321.803	4337.24	.5624	.5631	.6270	2.0774	1.3025	.0003003	
329.352	4352.90	.5637	.5664	.6306	2.0715	1.3063	.0003001	
337.578	4369.95	.5661	.5709	.6357	2.0651	1.3128	.0003004	
340.558	4376.11	.5662	.5718	.6367	2.0628	1.3134	.0003001	
344.242	4383.71	.5668	.5734	.6384	2.0600	1.3151	.0003000	For λ = 4408.58 n-1 =.0003001
354.535	4404.95	.5700	.5792	.6449	2.0519	1.3232	.0003004	
356.323	4408.58	.5697	.5794	.6451	2.0505	1.3238	.0003001	
363.212	4422.74	.5713	.5828	.6489	2.0451	1.3271	.0003001	
365.530	4427.50	.5726	.5847	.6510	2.0433	1.3302	.0003004	
367.145	4430.79	.5713	.5838	.6500	2.0420	1.3273	.0002996	
λ_2 372.878	4442.52	.5736	.5876	.6543	2.0376	1.3332	.0003001	

t_1 =24.6°C; p_1 = 744.1 mm; angle of tilt of camera = 12°;   3rd order of grating.

In the above table column I gives the relative positions of the lines on the photographic plate; column II the corresponding wave-lengths as taken from tables in Kayser's Handbuch; column III the mean values of the shifts measured; column IV, the shifts corrected for the distance of the line from the center of the plate; column V, the shifts reduced to standard conditions; column VI, the dispersion in Ångström units per millimeter as measured on the photographic plate; column VII, the shifts in Ångström units, or rather the change in wave-lengths due to the change in pressure; and column VIII the values of (n - 1) as calculated by equation (1). The calculation of the index of refraction from one line alone is not very reliable, for the physical characteristics of the line may be such as to influence the results. For this reason the practice of determining the final values from a number of lines has been adhered to. This is shown in column IX. The values of n - 1 were reduced to one particular line by taking as nearly as possible the same number of evenly spaced lines on each side of a central line, and taking the mean of the values. This practice is permissible in the region of the spectrum where the dispersion is comparatively small, but in the ultra-violet the separate values of n - 1 were reduced to one particular line by the use of Kayser and Runge's dispersion formula and the mean of these reduced values

taken as a final value. The table given above represents one of the better sets of observations. In some cases values of n - 1 had to be determined from a less number of lines.

Since the temperature correction for the shifts was not determined it was necessary to select some iron line whose index of refraction for air has been well determined by other observers, and refer all values to this line. For this purpose the line λ = 4315.29 was selected. The index of refraction was calculated from the dispersion formulae of Kayser and Runge(1), Rentschler(2) and Miss Howell(3), the values obtained for n - 1 being .0002961, .0002959 and .0002958. The mean of these is .0002959. The value obtained in the present investigation was .0003005, thus giving a difference of .0000046 which will be called the temperature correction. From the following discussion it will be seen that this correction is constant for all regions of the spectrum; and the corrected values of the index of refraction for air for all wave-lengths is found by subtracting this correction from the computed values.

The formula for the Littrow type of mounting of the plane grating is

$$m\lambda = a (\sin \theta + \sin i)$$

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(1) Kayser and Runge "Ann d. Physik" (50), 293, 1893.
(2) Rentschler, "Astrophysical Jl." 28, 345, 1908.
(3) Miss Howell, Physical Review, VI, 81, 1915.

where m is the order of the spectrum, λ the wave-length of the spectrum line, a the grating space, θ the angle the incident light makes with the normal to the grating, and i the angle of the refracted beam. For the present discussion θ and i can be considered as remaining constant. Suppose a temperature change in the grating occurs, then the expansion produces a change, δa, in the grating space. The corresponding change in the wave-length, $\delta\lambda$, is given by the relation

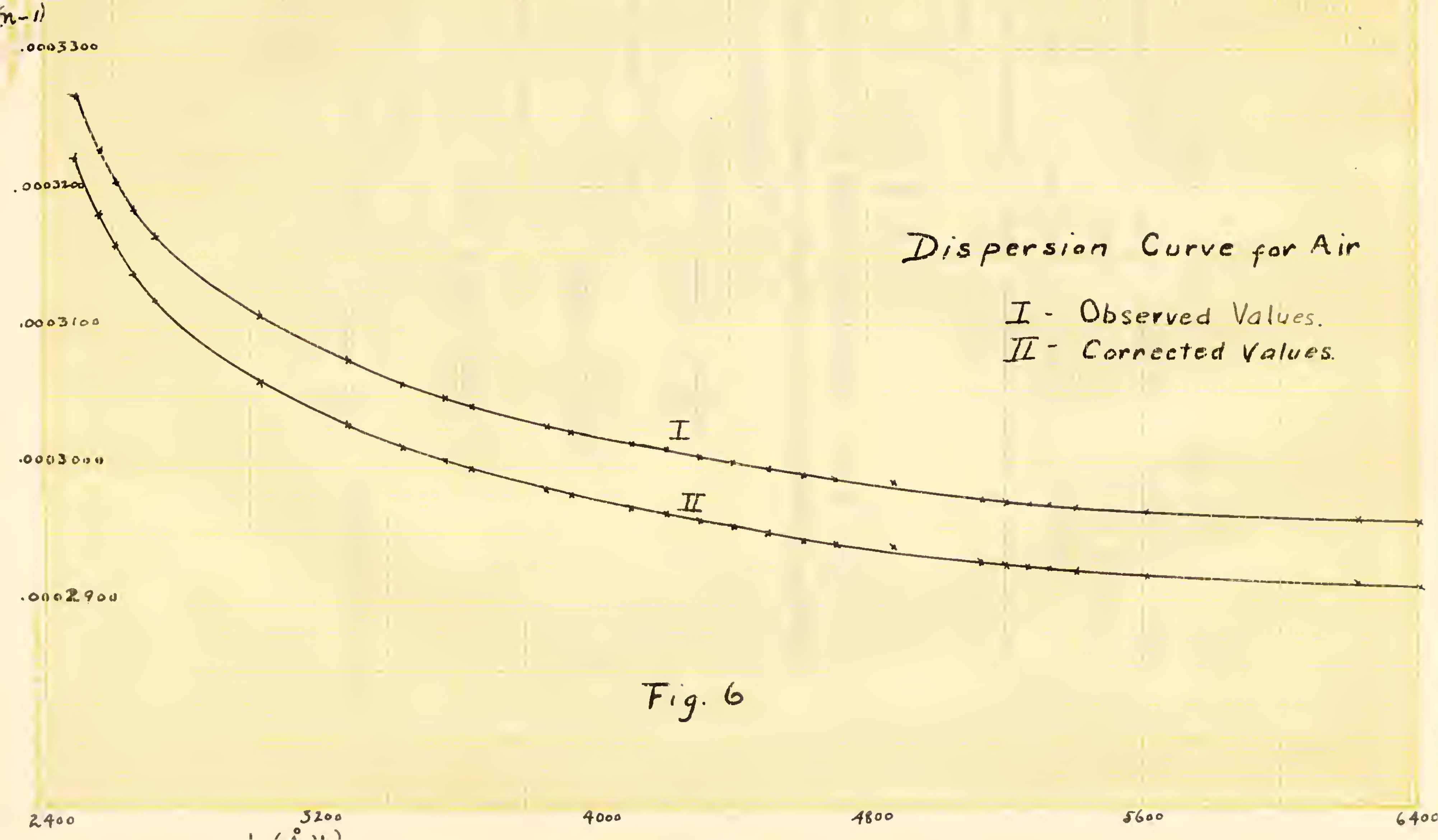
$$m\delta\lambda = \delta a \left(\sin\theta + \sin i \right)$$

Dividing this expression by the one above, we have

$$\frac{\delta\lambda}{\lambda} = \frac{\delta a}{a} = \text{constant},$$ if the change δa is a constant in every case. The exposures were carried out under exactly the same conditions so that δa would be a constant. Therefore the correction to n - 1 for temperature changes is a constant for all values of λ.

The indices of refraction of air for the different wave-lengths obtained in this manner are given in the following table. The first column contains the values of the wave-lengths, the second, the observed values of n - 1, and the third the corrected values of n - 1. In the fourth column are recorded the number of lines observed in determining n - 1 for the specified wave-length, λ.

λ (A.U.)	n-1 (observed)	n-1 (corrected)	No. lines used in determining (n-1)
6411.90	.0002956	.0002910	3
6230.93	.0002960	.0002914	4
5615.89	.0002965	.0002919	8
5406.02	.0002968	.0002922	8
5328.21	.0002971	.0002925	4
5269.70	.0002971	.0002925	3
5202.49	.0002972	.0002926	9
5139.64	.0002975	.0002929	1
4879.39	.0002986	.0002940	7
4710.46	.0002988	.0002942	2
4618.95	.0002991	.0002945	7
4517.70	.0002997	.0002951	9
4408.56	.0003001	.0002955	7
4315.29	.0003005	.0002959	12
4219.52	.0003011	.0002965	7
4118.70	.0003015	.0002969	18
3941.03	.0003023	.0002977	12
3867.35	.0003027	.0002981	10
3651.61	.0003043	.0002997	12
3575.60	.0003049	.0003003	3
3450.46	.0003058	.0003012	12
3292.73	.0003076	.0003030	11
3037.51	.0003107	.0003061	9
2727.64	.0003165	.0003119	14
2664.77	.0003184	.0003138	4
2613.91	.0003205	.0003159	11
2567.01	.0003228	.0003182	8
2493.34	.0003268	.0003222	4



Dispersion Curve for Air

I - Observed Values.
II - Corrected Values.

Fig. 6

The dispersion curve is shown in Fig. 6. Curve I indicates the values obtained from the observed values of the shifts of the lines on the photographic plate, while curve II represents the values of n - 1 referred to the known value for the line $\lambda = 4315.29$.

It is customary to express dispersion curves in terms of the Cauchy formula

$$n - 1 = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$$

although this is of no theoretical importance. It is given here so that the results can be compared with those of other observers. According to Cuthbertson(1)

$$n - 1 = 10^{-7} \left(2885.4 + \frac{13.38}{\lambda^2} + \frac{.50}{\lambda^4}\right);$$

Kayser and Runge(2) find

$$n - 1 = 10^{-7} \left(2878.7 + \frac{13.16}{\lambda^2} + \frac{.316}{\lambda^4}\right);$$

Rentschler(3) finds

$$n - 1 = 10^{-7} \left(2903 + \frac{3.80}{\lambda^2} + \frac{1.23}{\lambda^4}\right);$$

Miss Howell(4) finds

$$n - 1 = 10^{-7} \left(2881.7 + \frac{11.83}{\lambda^2} + \frac{.46}{\lambda^4}\right)$$ where λ is expressed

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(1) Loc. cit.
(2) " "
(3) " "
(4) " "

in thousandths of a millimeter. In the present investigation the values of the constants, a, b and c, in the Cauchy formula were obtained by the method of least mean squares, and the following formula was found

$$n - 1 = 10^{-7} \left(2878.12 + \frac{11.59}{\lambda^2} + \frac{.551}{\lambda^4} \right)$$

This is in good agreement with the formula of Kayser and Runge and that of Miss Howell.

Discussion of Results.

In plotting the values of (n - 1) against the wave-lengths , it was found that a remarkably smooth curve could be traced through the points. In only one case did a point fail to touch this curve. This point is the one corresponding to λ = 4879.39 and the reason for this can be found in the physical characteristics of the lines in this region. Apparently they are broad and not well-defined, and accurate settings on them were almost impossible. As the iron arc gives sharper lines in the violet and ultra-violet it is safe to say that the values below λ = 4500 are more accurate.

The magnitude of the shift is also a determining factor in the accuracy of the results as well as the sharpness of the lines. As dispersion in a grating increases with the number of the order of spectrum used, more accurate results can be obtained with the higher

orders provided the intensity of the spectrum line does not decrease too rapidly. The third order was used above λ = 4300 and the shift taken from the plate varied from .55 mm. to .92mm. From λ = 4300 to λ = 3000 the fourth order was used the measured shifts varying from .76mm. to .52 mm. Below λ = 2800 the third order was used, the measured shifts being of the order of magnitude of .45 mm. In this last case the focus of the lens changed rapidly and the photographic plate had to be tilted so that its normal made a large angle with the axis of the lens. This increased the dispersion and it accounts for the large value of the measured shifts even though the third order of spectrum was used. Thus it is seen that even in the extreme ultra-violet the accuracy, so far as the magntidue of the shift is concerned, is about the same as that in the regions of longer wave-lengths.

The dispersion curve was extended as far as λ = 2493. The reason for not extending it farther is found in the arrangement of the apparatus. The opening in the bell-jar was too small to allow the use of all the grating space and as this was limited by the dimensions of the quartz plate covering it no change could be made at that time. When ordering the quartz plate this point was over-

looked. The intensity, consequently, was greatly reduced and longer exposures than those used were found undesirable on account of the temperature changes in the grating.

From a consideration of a number of separate measurements which are associated with a single value of n - 1 and the good agreement of these measurements, it is safe to say that the fourth significant figure is determined to a fair degree of accuracy. It can be relied upon to be correct to within ~~several~~ a few units.

As far as the method is concerned there seems to be only one serious difficulty, - the effect of temperature changes as the pressure of the gas is changed. It is believed that, with more time and several improvements in the present apparatus, the value of this temperature correction can be determined and consequently the absolute values of n - 1. Any change which will shorten the time of exposure and increase the dispersion without a loss to the sharpness of the spectrum line will give a greater accuracy to the results. This can be done by increasing the amount of grating space used, by the use of a grating with bright, higher orders, and the substitution of a lens of longer focal length, keeping the ratio of aperture to focus the same. The ratio in the present arrangement is about 1/50. The author hopes to continue this work in the near future, using

other gases besides air. In the present form this method is applicable to gases which do not injure the grating and for which a single value of the index of refraction is known.

Among the advantages of this method the following may be mentioned: First, any source of light may be used which gives fairly sharp lines, the number being immaterial provided there are enough in any one region to determine the dispersion; second, the final value of n - 1 is not obtained from observations on a single line; third, values of n - 1 can be obtained for wave-lengths of all lines that can be photographed in a reasonable length of time using an ordinary grating spectrometer. This last fact makes the method a very desirable one in extending into the extreme ultra-violet the dispersion curves of gases.

Summary.

1. The application of the plane grating to the determination of the index of refraction of gases has been thoroughly studied and the only serious difficulty found to be one due to temperature changes.

2. The dispersion curve of air has been obtained from $\lambda = 6412$ to $\lambda = 2493$.

3. The accuracy of the corrected values of the index of refraction of air agree with those obtained by other methods.

In conclusion I wish to thank Dr. Anderson, who suggested the present problem and whose interest, help and constant encouragement have been invaluable.

I am indebted to Professor Ames for his interest and advice throughout the course of the work. I wish also to thank Dr. Pfund for his advice and helpful suggestions.

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Biographical Sketch.

Robert William Dickey, son of Robert J. and Mattie (Jones) Dickey, was born May 13, 1891 in Bath County, Virginia, and was prepared for college in the High School of Covington, Va. In 1906 he entered Washington and Lee University, receiving the degrees of B.S., B.A., and M.A. in the years 1910, 1911 and 1912 respectively. While there he was assistant in Physics for five years besides holding the Physics and Civil Engineering Scholarships, the Bradford Scholarship and the Howard Houston Fellowship for two years. During the year 1912-13 he was instructor in Physics and Mathematics in Washington and Lee University. In 1913 he entered the Johns Hopkins University as a graduate student in Physics, and has been a holder of a Virginia Scholarship (1913-1914), assistant in Physics (1914-1915) and Fellow in Physics (1915-1916). In 1915 he was admitted to the degree of Master of Arts. He has pursued courses of instruction under Professors Ames, Wood, Whitehead, Anderson and Pfund in Physics, Applied Electricity and Astronomy, and Dr. Cohen in Mathematics.